January Assignment: Part 2

Topics on Games

2024-01-15

Problem 1. Total: 4

The Game of Sim is a pen-and-paper game played on a complete graph on n vertices, traditionally $n = 6$. It's a two-player game.

Initially, all edges of the graph are black or uncolored. The players take turns coloring any uncolored lines.

One player colors in one color, and the other colors in another color, with each player trying to avoid the creation of a triangle made solely of their color; the player who completes such a triangle loses immediately.

You can play this game here.

(1.a) (1 point) Consider the case when $n = 4$. Explain why both players have a drawing strategy.

(1.b) (1 point) Consider the case when $n = 5$. Explain why both players have a drawing strategy.

(1.c) (1 point) Consider the case when $n = 6$. Explain why the game *cannot* end in a draw.

(1.d) (1 point) Consider the case when $n = 6$. Recall that we used a "strategy stealing" argument to establish that there exists a winning strategy for the first player in Chomp. Explain why a similar argument fails here.

Problem 2. Total: 2

Lata and Raj are playing a game called *Keep and Split*. They start with two piles of coins, one of size M and one of size K. On a player's turn, s/he takes away one of the piles entirely and splits the other pile into two (non-empty) piles any way s/he wants. Whoever can't make a legal move (i.e, is presented with two piles of one coin each) loses. Note that taking as many coins as possible is not the object of the game, though it may be a pleasant side effect.

(2.a) (1 point) Raj gets to decide whether to go first or second. What should he do when $M = 2018$ and $K = 2019$?

(2.b) (1 point) Describe a general condition that you can use to determine if (M, K) is a winning state for the first player.

Problem 3. Total: 6

Lata and Raj are still playing *Keep and Split*. Suppose Lata and Raj now have three different types of coins. The game is now played with six piles of coins – two piles of each type. On a player's turn, he chooses a type of coins, takes away one of the piles of that type, and splits the other. Whoever can't move loses. If the numbers are $(M1, K1)$, $(M2, K2)$ and $(M3, K3)$, we want to understand: is it better to go first or second, and what is the winning strategy?

(3.a) (1 point) Fill out the a 8×8 table of nimvalues for *Keep and Split*. The (M,K) entry of your table should show the nimvalue of the game with piles of size M and K.

For example, the values for Row 1 are: `[0,1,0,2,0,1,0,3]`.

(3.b) (1 point) The game is $(2, 4)$, $(1, 8)$, and $(3, 5)$. It is Raj's turn. What should he do? Explain how to derive the answer from your table.

(3.c) (1 point) Describe a formula (or algorithm) for computing the nimvalue of (M, K) for all M and K.

(3.d) (2 points) Argue why your proposed formula/algorithm works.

(3.e) (1 point) The game is $(2, 4)$, $(1, 8)$, and $(2024, 2023)$. It is Raj's turn. What should he do? Explain how to derive the answer from your formula or pattern (even if you haven't proved it).

Problem 4. Total: 3

In the game of *Corner A Piece*, we are given a chess piece on a chess board that has been hit by a diagonal plague, so it can only move south, west, and south-west.

Initially, we are given:

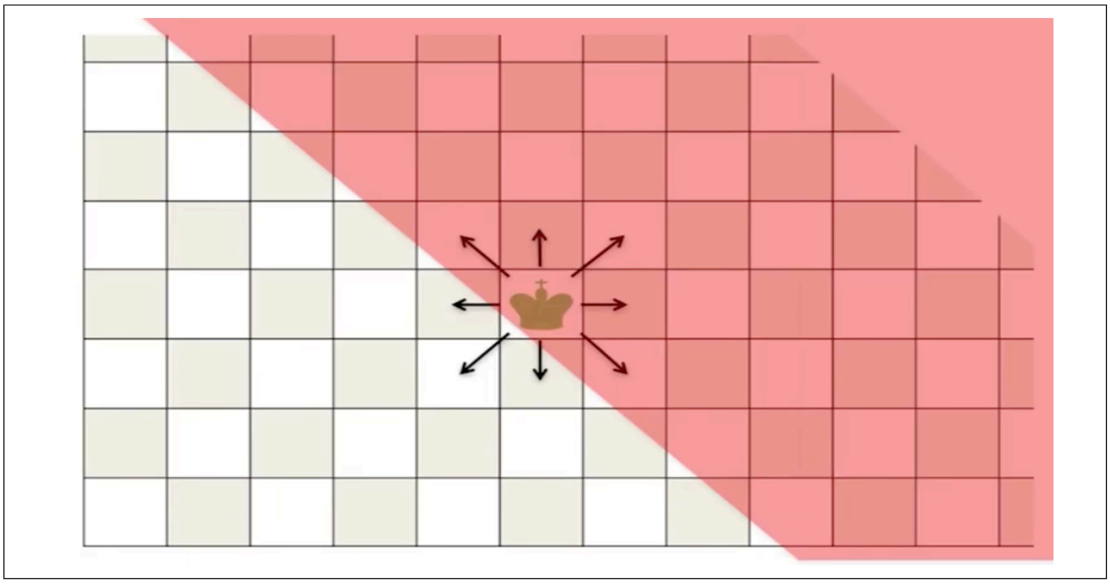
- a chess piece (e.g, rook, queen, knight, king, etc.)
- an initial location for said piece (e.g, (5,3))

We have two players who take turns to play. On their turn, the player can move the piece from its current location to any location using a valid move that moves the piece in a south, west, or south-west direction. Note that once the piece has reached the bottom-left corner square, there are no legal moves left.

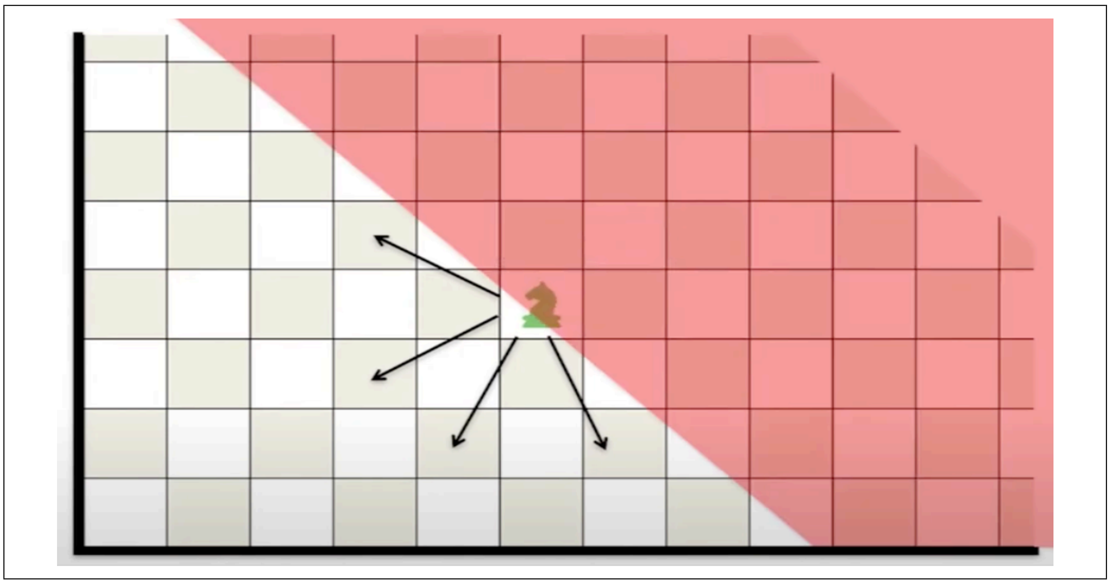
The player who cannot move loses.

(4.a) (1 point) Characterize the squares from which the first player can win if the piece is a king.

Note that the legal moves are given by the following image:



(4.b) (2 points) Characterize the squares from which the first player can win if the piece is a knight.



Question	Points	Score
Problem 1	4	
Problem 2	2	
Problem 3	6	
Problem 4	3	
Total:	15	

 End of the exam