CHAPTER FOUR

Ticktacktoe

WHO HAS NOT as a child played ticktacktoe, that most ancient and universal struggle of wits of which Wordsworth wrote (*Prelude,* Book I):

> *At evening, when with pencil, and smooth slate*
> *In square divisions parcelled out and all*
> *With crosses and with cyphers scribbled o'er,*
> *We schemed and puzzled, head opposed to head*
> *In strife too humble to be named in verse.*

At first sight it is not easy to understand the enduring appeal of a game which seems no more than child's play. While it is true that even in the simplest version of the game the number of possible moves is very large—15,120 ($9 \times 8 \times 7 \times 6 \times 5$) different sequences for the first five moves alone—there are really only a few basic patterns, and any astute youngster can become an unbeatable player with only an hour or so of analysis of the game. But ticktacktoe also has its more complex variations and strategic aspects.

In the lingo of game theory, ticktacktoe is a two-person contest which is "finite" (comes to a definite end), has no element of chance and is played with "perfect information," all moves being known to both players. If played "rationally" by both sides, the game must end in a draw. The only chance of winning is to catch an unwary opponent in a "trap" where a row can be scored on the next move in two ways, only one of which can be blocked.

Of the three possible opening plays— a corner, the center or a side box— the strongest opening is the corner, because the opponent can avoid being trapped at the next move only by one of the eight possible choices: the center. Conversely, center opening traps can be blocked only by seizing a corner. The side opening, in many ways the most interesting because of its richness in traps on both sides, must be met by taking one of four cells. The three openings and the possible responses by a second player who plays rationally are diagramed in Figure 17.

Variants of ticktacktoe more exciting mathematically than the present form were played many centuries before

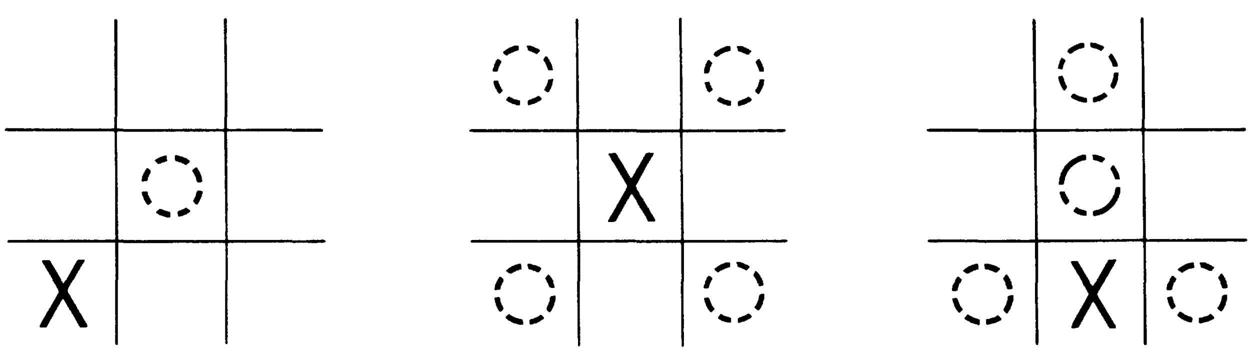

FIG. 17.
The first player (X) has a choice of three openings. To avoid losing, second player (O) must choose one of the cells indicated.

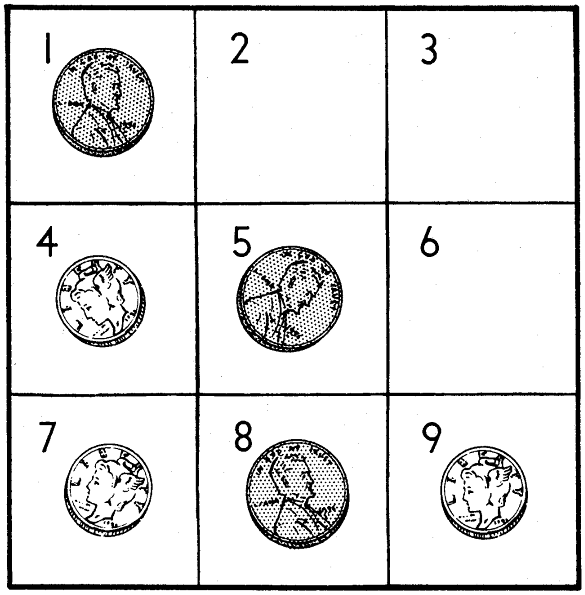

FIG. 18.
Ticktacktoe with moving counters.

the Christian era. All of them employ six counters and can be played on the board pictured in Figure 18— one player using three pennies, the other, three dimes. In the simplest form, popular in ancient China, Greece and Rome, players take turns placing a counter on the board until all six are down. If neither player has won by getting three in a row (orthogonally or diagonally) they continue playing by moving on each turn a single counter to any adjacent square. Only moves along the orthogonals are permitted.

Ovid mentions this game in Book III of his *Art of Love,* including it among a group of games which he advises a woman to master if she wishes to be popular with men. The game was common in England in 1300 when it was called "three men's morris," the ancestor of nine, eleven, and twelve men's morris, or "mill" as it is usually called in the United States today. Since the first player has a sure win by playing first in the center, this opening is usually barred. With this restriction the game is a draw if played rationally, but it swarms with potential traps on both sides.

A variation of this game permits moves to neighboring cells along the two main diagonals of the square. A further extension (attributed to early American Indians) allows any counter to move one step in any direction, orthogonally or diagonally (e.g., a move can be made from cell 2 to cell 4). In the first version the initial player can still force a win if allowed to open on the center, but the second variant is probably a draw. A free-wheeling version called *les pendus* (the hanged) in France, permits any piece to be moved to *any* vacant cell. This also is believed drawn if played rationally.

Many variations of moving-counter ticktacktoe have been applied to 4 x 4 boards, each player using four counters and striving to get four in a row. A few years ago magician John Scarne marketed an interesting 5 x 5 version called "teeko." Players take turns placing four counters each, then alternate with one-unit moves in any direction. A player wins by getting four in a row, orthogonally or diagonally, or in a square formation on four adjacent cells.

Many delightful versions of ticktacktoe do not, however, make use of moving counters. For example: toetacktick (a name supplied by reader Mike Shodell, of Great Neck, New York). This is played like the usual game except that the first player to get three in a row *loses*. The second player has a decided advantage. The first player can force a draw only if he plays first in the center. Thereafter, by playing symmetrically opposite the second player, he can insure the draw.

In recent years several three-dimensional ticktacktoe games have been marketed. They are played on cubical boards, a win being along any orthogonal or diagonal row as well as on the four main diagonals of the cube. On a 3 x 3 x 3 cube the first player has an easy win. Curiously, the game can never end in a draw because the first player has fourteen plays and it is impossible to make all fourteen of them without scoring. The 4 x 4 x 4 cube leads to more interesting play and may or may not be a draw if played rationally.

Other ways of playing on cubes have been proposed. Alan Barnert of New York suggests defining a win as a square array of counters on any of the orthogonal planes as well as on the six main diagonal planes. Price Parks and Robert Satten, while students at the University of Chicago in 1941, devised an interesting 3 x 3 x 3 cubical game in which one wins by forming two intersecting rows. The winning move must be on the point of intersection. Because an early move into the center cubicle insures a win, this move is barred unless it is a winning move or necessary to block an opponent from winning on his next move.

Four-dimensional ticktacktoe can be played on an imaginary hypercube by sectioning it into two-dimensional squares. A 4 x 4 x 4 x 4 hypercube, for example, would be diagramed as shown in Figure 19. On this board a win of four in a row is achieved if four marks are in a straight line on any cube that can be formed by assembling four squares in serial order along any orthogonal or either of the two main diagonals. Figure 20 shows a win on such an assembled cube. The first player is believed to have a sure win, but the game may be a draw if played on a 5 x 5 x 5 x 5 hypercube. The number of possible rows on which one can win on a cube of n-dimensions is given by the following formula (n is the number of dimensions, k the number of cells on a side):

$$\frac{(k+2)^n - k^n}{2}$$

For an explanation of how this formula is derived, see Leo Moser's comments in the *American Mathematical Monthly*, February 1948, page 99.

The ancient Japanese game of *go-moku* (five stones), still popular in the Orient, is played on the intersections of a *go* board (this is equivalent to playing on the cells of a 19 x 19 square). Players take turns placing counters from an un-

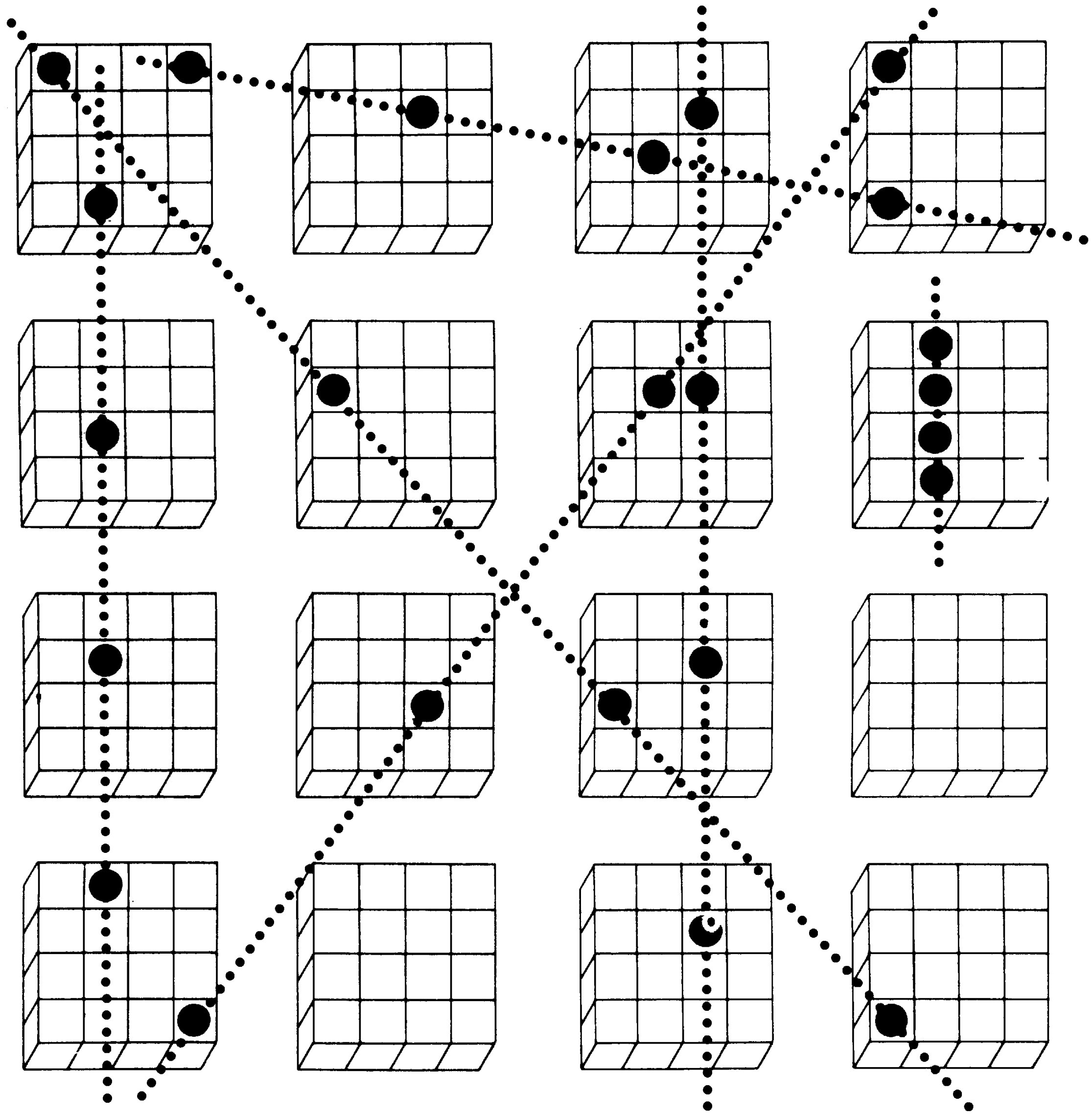

FIG. 19.
Four-dimensional ticktacktoe. Dotted lines show some winning plays.

limited supply until one player wins by getting five in a line, orthogonally or diagonally. No moves are allowed. Experts are of the opinion that the first player can force a win, but as far as I know, no proof of this has ever been published. The game became popular in England in the 1880s under the name of "go-bang." It was sometimes played on an ordinary checker board, each player using 12 or 15 checkers. Moves were permitted in any direction if no one had won by the time all the checkers were placed.

During the past decade a number of electrical ticktacktoe playing machines have been constructed. It is interesting to learn that the first ticktacktoe robot was invented (though never actually built) by Charles Babbage, the nineteenth-century English pioneer inventor of calculating devices. Babbage planned to exhibit his machine in London to raise funds for more ambitious work, but abandoned his plans after learning that current London exhibits of curious machines (including a "talking machine" and one that made Latin verses) had been financial flops.

A novel feature of Babbage's robot was its method of ran-

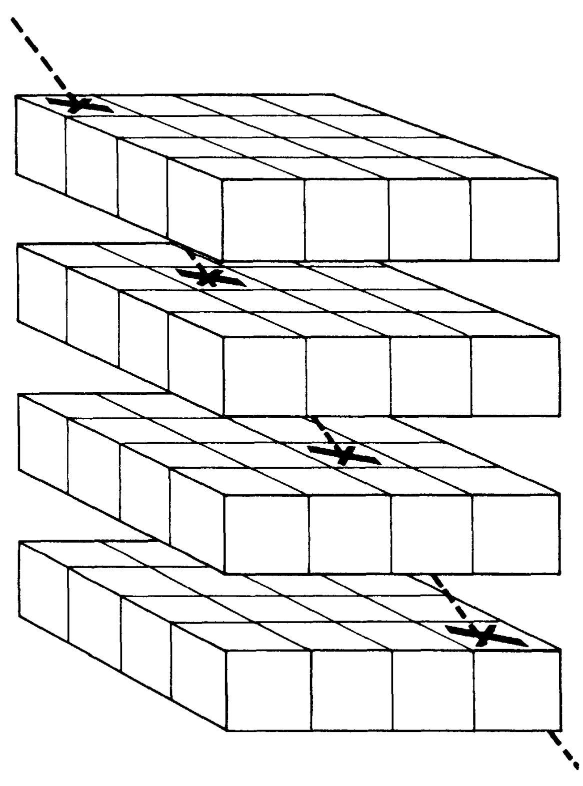

FIG. 20. The assembled cube.

domizing choices when faced with alternate lines of equally good play. The machine kept a running total of the number of games won. If called upon to choose between moves A and B, the machine consulted this total, played A if the number was even, B if odd. For three alternatives, the robot divided the total by 3 to obtain a remainder of 0, 1 or 2, each result gearing it to a different move. "It is obvious that any number of conditions might be thus provided for," Babbage writes in his *Passages from the Life of a Philosopher,* 1864, pages 467-471. "An inquiring spectator . . . might watch a long time before he discovered the principle upon which it [the robot] acted."

Unfortunately Babbage left no record of what he calls the "simple" mechanical details of his machine, so one can only guess at its design. He does record, however, that he "imagined that the machine might consist of the figures of two children playing against each other, accompanied by a lamb and a cock. That the child who won the game might clap his hands whilst the cock was crowing, after which, that the child who was beaten might cry and wring his hands whilst the lamb began bleating." A less imaginative ticktacktoe machine, displayed in 1958 at the Portuguese Industrial Fair in Lisbon, cackled when it won, snarled when (presumably set on a "poor play" circuit) it lost.

It might be thought that programing a digital computer to play ticktacktoe, or designing special circuits for a ticktacktoe machine, would be simple. This is true unless your aim is to construct a master robot that will win the maximum number of games against inexperienced players. The difficulty lies in guessing how a novice is most likely to play. He certainly will not move entirely at random, but just how shrewd will he be?

To give an idea of the sort of complications that arise, assume that the novice opens on cell 8. The machine might do well to make an irrational response by seizing cell 3! This would be fatal against an expert, but if the player is only moderately skillful, he is not likely to hit on his one winning reply, cell 9. (See comments on Alain White's article in the bibliography.) Of the six remaining replies, four are disastrous. There will be, in fact, a strong temptation for him to play on cell 4 because this leads to two promising traps against the robot. Unfortunately, the robot can spring its own trap by following with cell 9, then 5 on its next move. It might turn out that in actual play the machine would win more often by this reckless strategy than with a safe course that would most likely end in a draw.

A truly master player, robot or human, would not only know the most probable responses of novices, as determined by statistical studies of past games; he would also analyze each opponent's style of play to determine what sort of mistakes the opponent would most likely make. If the novice improved as he played, this too would have to be considered. At this point the humble game of ticktacktoe plunges us into far from trivial questions of probability and psychology.

ADDENDUM

THE NAME "ticktacktoe" has many variations in spelling and pronunciation. According to the *Oxford Dictionary of Mother Goose Rhymes,* 1951, page 406, it derives from an old English nursery rhyme that goes:

Tit, tat, toe,
My first go,
Three jolly butcher boys all in a row.
Stick one up, stick one down,
Stick one in the old man's crown.

I have observed that many ticktacktoe players are under the mistaken impression that because they can play an unbeatable strategy they have nothing more to learn about the game. A master player, however, must be quick to take the best possible advantage of a bad play. The following three examples, all from the side opening, will make this clear.

If you open with X8 and he follows with O2, your best response against a novice is X4 because it wins in four out of six moves now open to O. He can block your traps only by playing O7 or O9.

If he opens with X8 and you respond with a lower corner, say O9, you can spring winning traps if he plays X2, X4 or X7.

If he opens with X8, a response of O5 may lead to an amusing development. Should he take X2, you can then permit him to designate your own next move for it is impossible for you to play without being able to set a winning trap!

It was mentioned in the chapter that the moving counter variation popular in ancient Rome is a win for the first player if he takes the center square. For readers who are interested, the two possible lines of forced play are as follows:

	X	O
	5	3
	4	6
(1)	9	1
	4 *to* 7	*Any move*
	5 *to* 8	
	5	6
	1	9
(2)	3	2
	1 *to* 4	*Any move*
	4 *to* 7	

These lines of play will win regardless of whether moves along the two main diagonals are or are not permitted, but the first one fails if moves along short diagonals are legal.