On the d-Runaway Rectangle Escape Problem

Aniket Basu Roy, Sathish Govindarajan, Neeldhara Misra, Shreyas Shetty

Computer Science & Automation,
Indian Institute of Science, Bangalore

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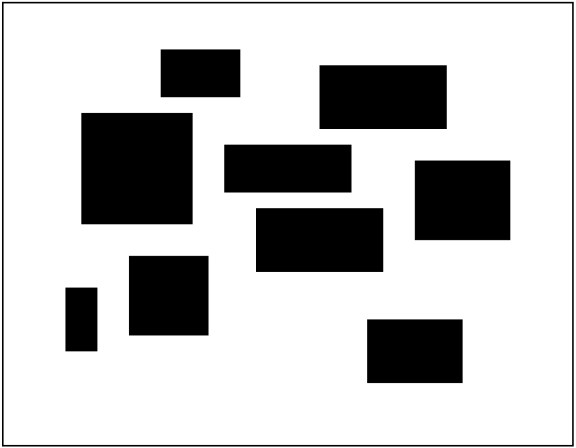
Rectangle Escape Problem



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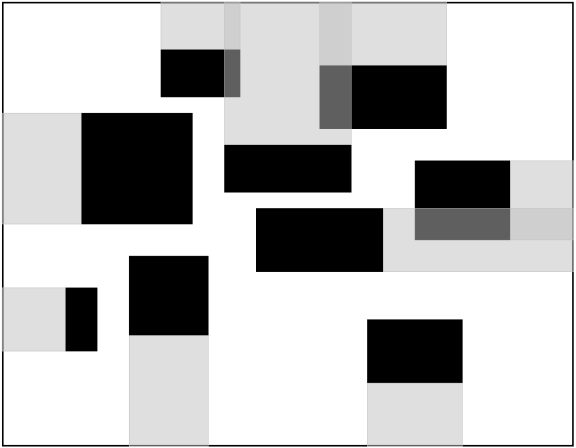

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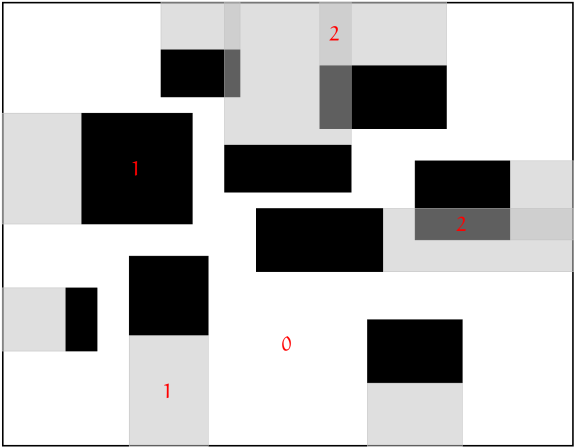

$\mathcal{R} \triangleq$ family of rectangles
$\tilde{\mathcal{R}} \triangleq$ family of rectangles post extension
$\text{density}(p) \triangleq$ the number of rectangles containing point p
$\text{density}(\tilde{\mathcal{R}}) \triangleq \max_{p} \text{density}(p)$

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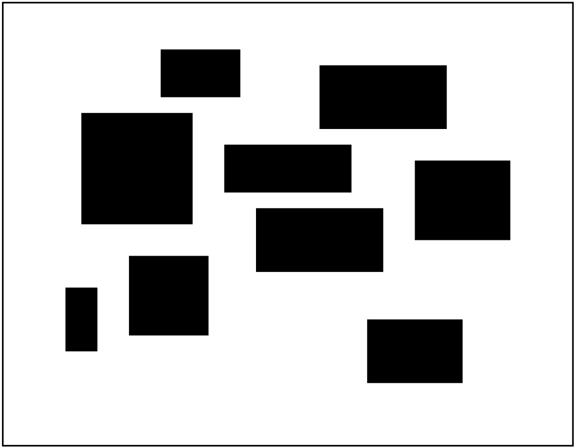
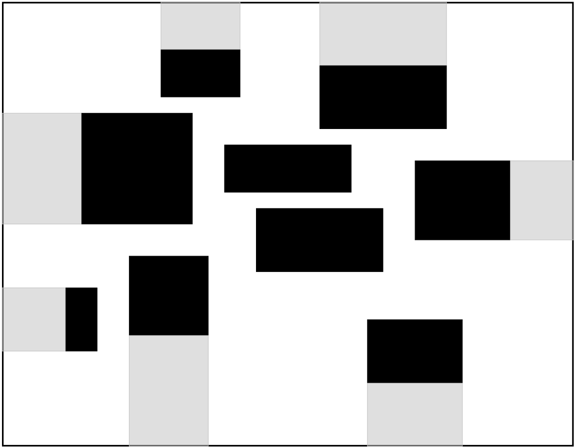
Definition

Given $\mathcal{R}$, minimize $\text{density}(\tilde{\mathcal{R}})$ such that for every $R \in \mathcal{R}$, R is escaped in one of the four directions.

Sepehr Assadi, Ehsan Emamjomeh-Zadeh, Sadra Yazdanbod, and Hamid Zarrabi-Zadeh.
On the Rectangle Escape Problem.
In *Canadian Conference on Computational Geometry (CCCG)*, 2013.

- NP-Hard for $d \geq 2$
- $O(n^4)$ time algorithm for $d = 1$
- Inapproximability within factor of $\frac{3}{2}$
- Randomized $(1 + \epsilon)$-approximation algorithm with high probability when optimal answer is $\Omega(\ln n)$





d-Runaway Rectangle Escape Problem

Definition

Given $\mathcal{R}$ and d, maximize the number of rectangles that can escape in one of the four directions having density$(\tilde{\mathcal{R}}) \leq \mathrm{d}$.

- A $4(1+\frac{1}{d-1})$-approximation algorithm when rectangles are disjoint
- A $4d$-approximation algorithm
- NP-Hardness when the rectangles are unit squares from a grid
- An FPT algorithm
- W[1]-Hardness for 2-Constrained Runaway

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An Approximation Algorithm

- Approximation Factor: $4(1 + \frac{1}{d-1})$
- Assumption: Input Rectangles are disjoint

An Approximation Algorithm

$$\mathrm{OPT} = S_{\uparrow} \bigcup S_{\downarrow} \bigcup S_{\leftarrow} \bigcup S_{\rightarrow}$$

$$\exists \lambda \in \{\uparrow, \downarrow, \leftarrow, \rightarrow\}$$
$$\#S_{\lambda} \geq \frac{\#\mathrm{OPT}}{4}$$

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$\mathrm{OPT}_{\lambda} \triangleq$ optimum solution when $d-$RREP restricted to direction λ

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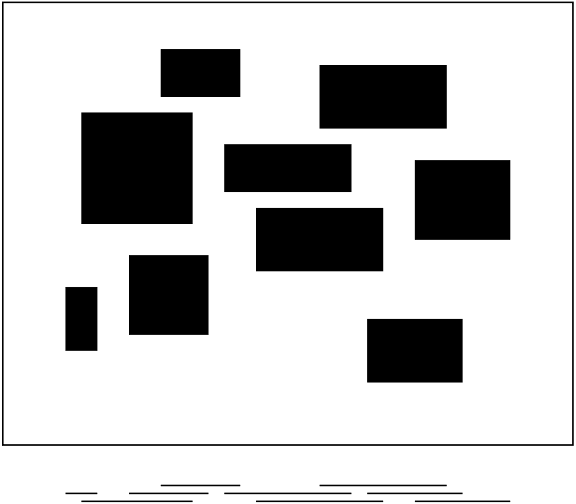

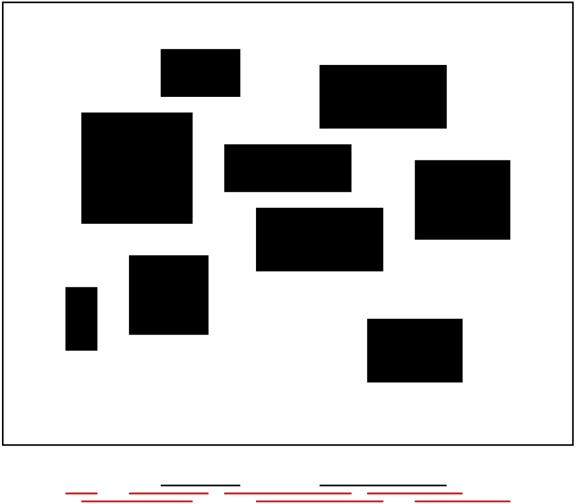

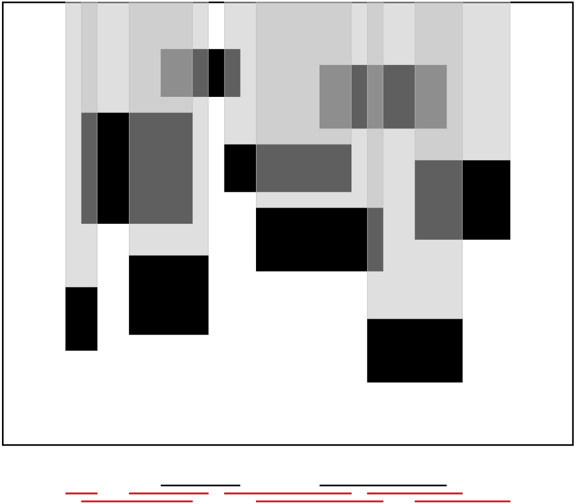

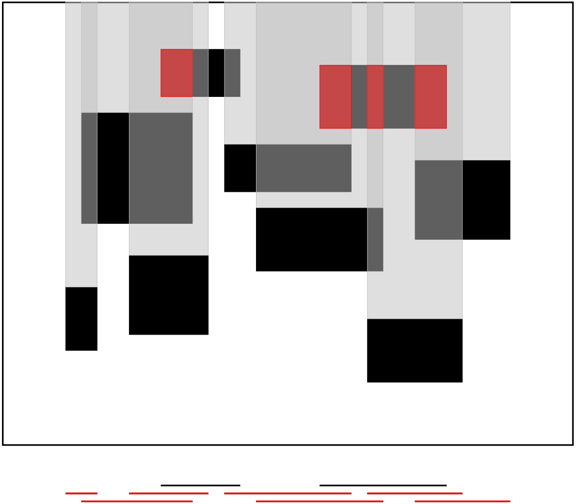

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An Approximation Algorithm

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$$\#\mathrm{PACK}_{d-1} \geq \#\mathrm{PACK}_{d}(1 - 1/d)$$

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$$\#PACK_{d-1} \geq \frac{\#OPT}{4(1 + \frac{1}{d-1})}$$

Another Approximation Algorithm

Approximation Factor: $4d$

Another Approximation Algorithm

$$\mathrm{OPT} = S_{\updownarrow} \bigcup S_{\leftrightarrow}$$

$$\exists \lambda \in \{\updownarrow, \leftrightarrow\}$$
$$\# S_{\lambda} \geq \frac{\#\mathrm{OPT}}{2}$$

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Another Approximation Algorithm

Stuck Rectangles

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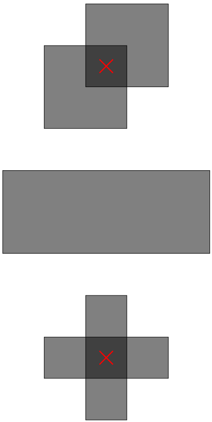
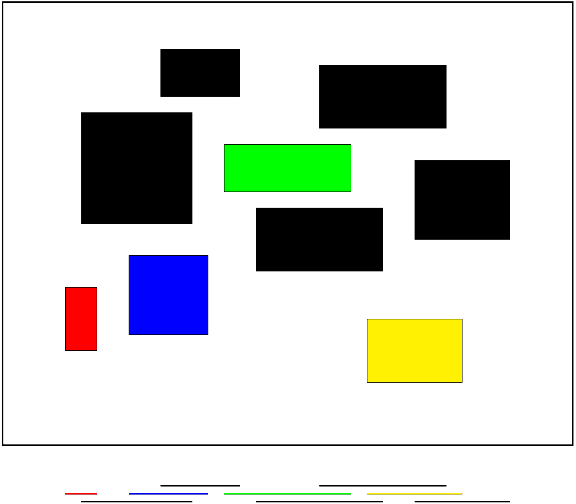
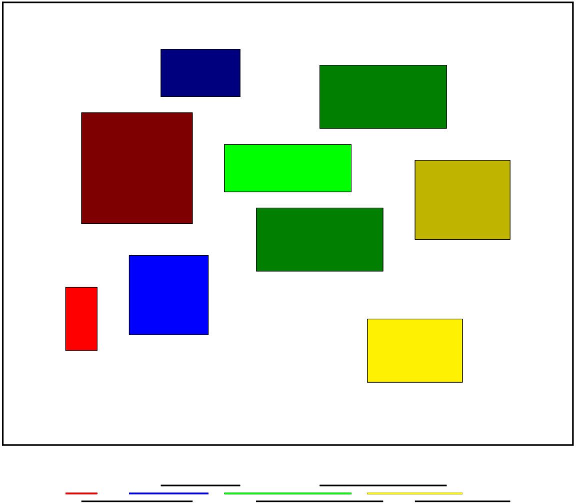
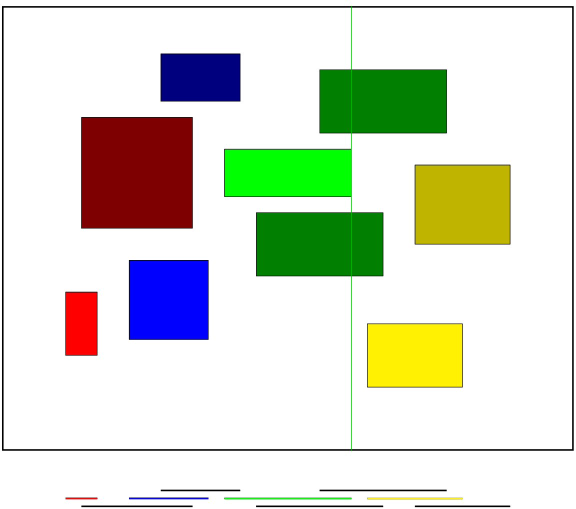


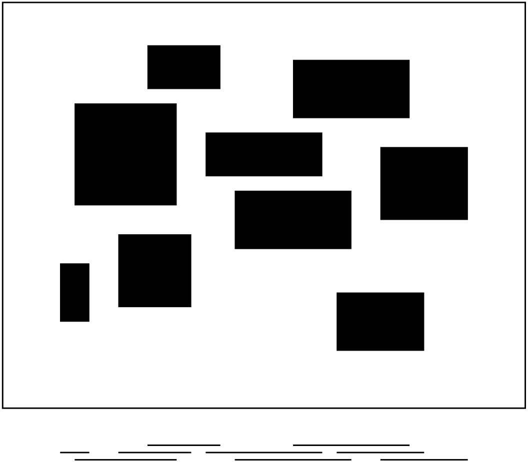


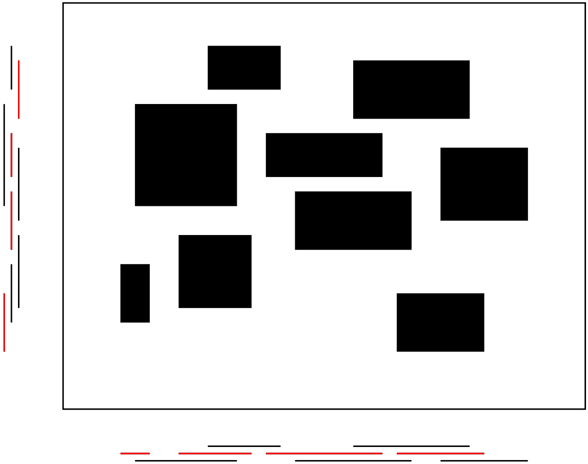




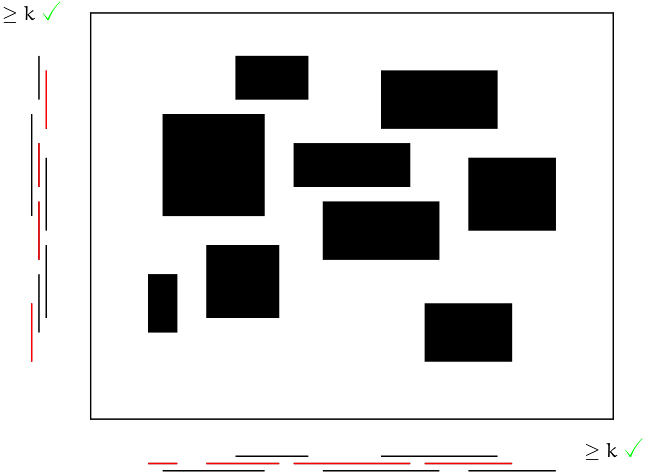
Given a set of rectangles $\mathcal{R}$, does there exist k rectangles that can escape without violating density d?

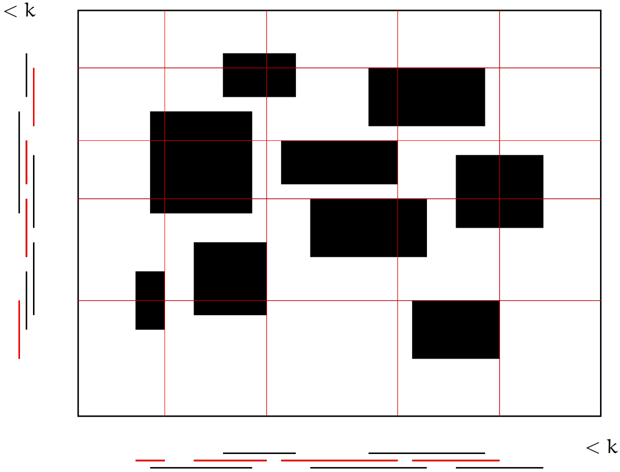
Assumption: Input density $\leq d - 1$





An FPT Algorithm





< k
< k

$$\#\text{Rectangles} \leq (d-1)(k-1)^2 < dk^2$$

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For every guess of k rectangles there are 4^k possibilities

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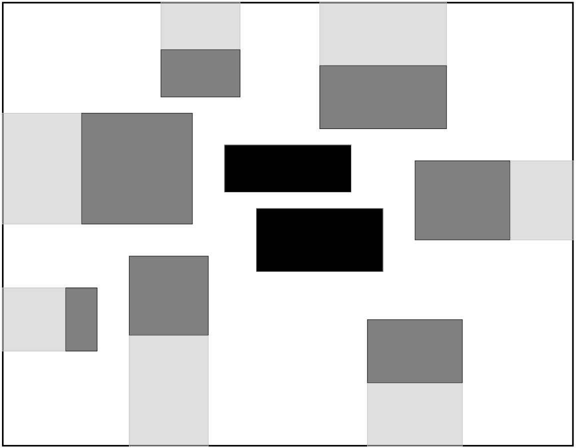
$$\text{Running time} = n^{O(1)} + 2^{O(k \log k)}$$



2-Constrained Runaway

$$\# \updownarrow \text{ extensions} \geq p$$
$$\# \leftrightarrow \text{ extensions} \geq q$$

2-Constrained Runaway

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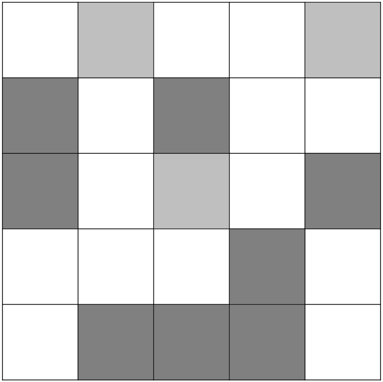
$\# \leftrightarrow$ extensions $\geq q$

Multi-Colored Clique $\leq_{\text{FPT}}$ 2-Constrained Runaway

$\Rightarrow$ 2-Constrained Runaway is W[1]-Hard

Square Escape Problem



Not All Equal LE3-SAT $\leq_P$ Square Escape Problem for density $= 2$

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$\Rightarrow$ Square Escape Problem (for density $= 2$) is NP-Hard

Open Problems

- Whether d-RREP admits a PTAS
- Better running time for the FPT algorithm viz. poly$(n, k)O(c^k)$
- Algorithms for Square Escape Problem
- Relation between k and d

Summary

- A $4(1 + \frac{1}{d-1})$-approximation algorithm when rectangles are disjoint
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