On the kernelization complexity of colorful motifs

IPEC 2010

joint work with

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Colorful Motifs.

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A problem with immense utility in bioinformatics.

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Intractable — from the kernelization point of view — on very simple graph classes.

An observation leading to many polynomial kernels in a very special situation.

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More observations ruling out approaches towards many poly kernels in more general situations.

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Some NP-hardness results with applications to discovering other hardness results.

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More observations ruling out approaches towards many poly kernels in more general situations.

Some NP-hardness results with applications to discovering other hardness results.

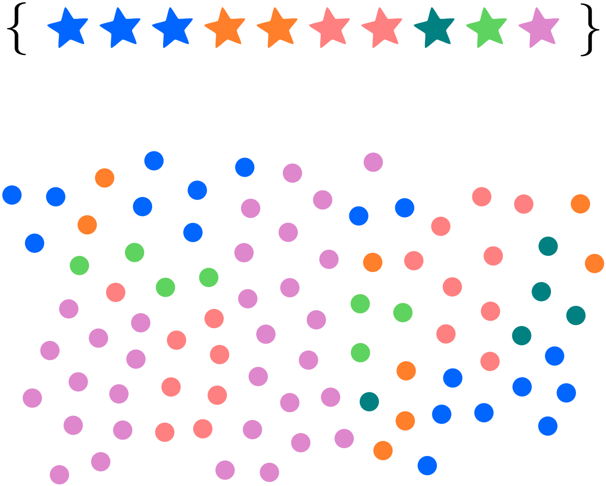
Observing hardness of polynomial kernelization on other classes of graphs.

Introducing Colorful Motifs

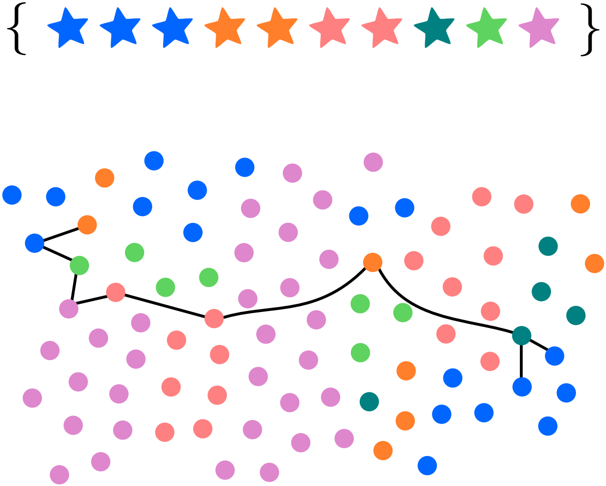
COLORFUL MOTIFS

Introducing Colorful Motifs

First: Graph Motifs



COLORFUL MOTIFS



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Topology-free querying of protein interaction networks.
Sharon Bruckner, Falk Hüffner, Richard M. Karp, Ron Shamir, and Roded Sharan.
In Proceedings of RECOMB 2009.

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Motif search in graphs: Application to metabolic networks.
Vincent Lacroix, Cristina G. Fernandes, and Marie-France Sagot.
IEEE/ACM Transactions on Computational Biology and Bioinformatics, 2006.

Sharp tractability borderlines for finding connected motifs in vertex-colored graphs.

Michael R. Fellows, Guillaume Fertin, Danny Hermelin, and Stéphane Vialette.

In Proceedings of ICALP 2007

COLORFUL MOTIFS

NP-Complete even when:

COLORFUL MOTIFS

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G is a tree with maximum degree 3.

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G is a bipartite graph with maximum degree 4 and M is a multiset over just two colors.

FPT parameterized by $|M|$,

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W[2]-hard parameterized by the number of colors in M.

Colorful Motifs

COLORFUL MOTIFS

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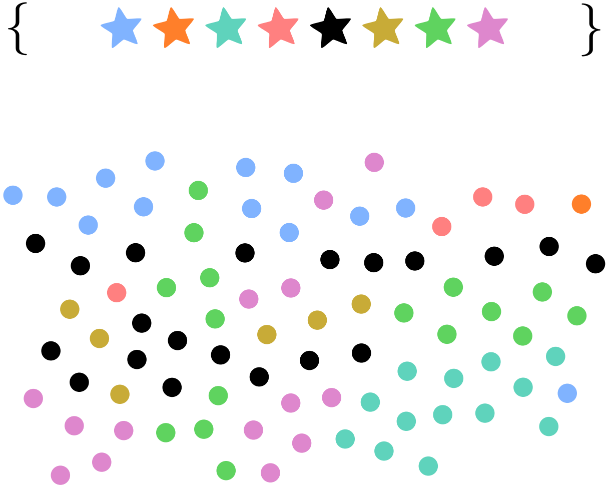

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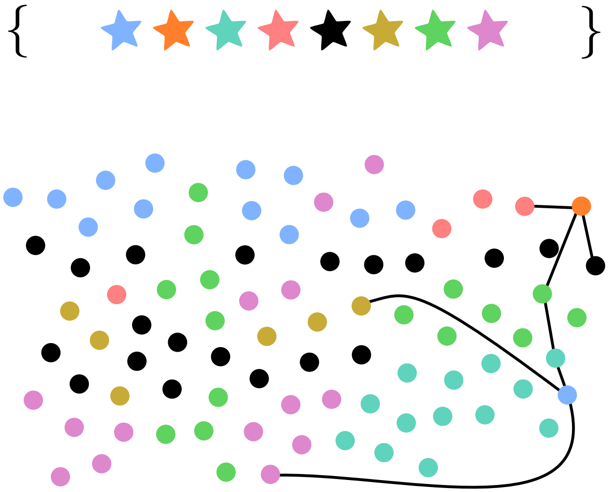

COLORFUL MOTIFS

Finding and Counting Vertex-Colored Subtrees.

Sylvain Guillemot and Florian Sikora.

In MFCS 2010.

COLORFUL MOTIFS

Finding and Counting Vertex-Colored Subtrees.

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A $\mathcal{O}^*(2^{|M|})$ algorithm.

COLORFUL MOTIFS

Kernelization hardness of connectivity problems in d-degenerate graphs.
Marek Cygan, Marcin Pilipczuk, Micha Pilipczuk, and Jakub O. Wojtaszczyk.
In WG 2010.

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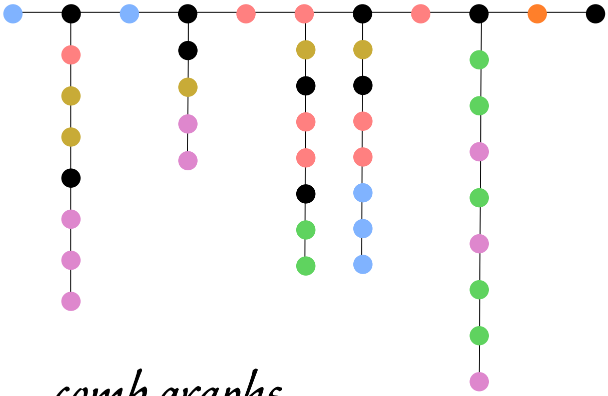
NP-complete even when restricted to....

COLORFUL MOTIFS

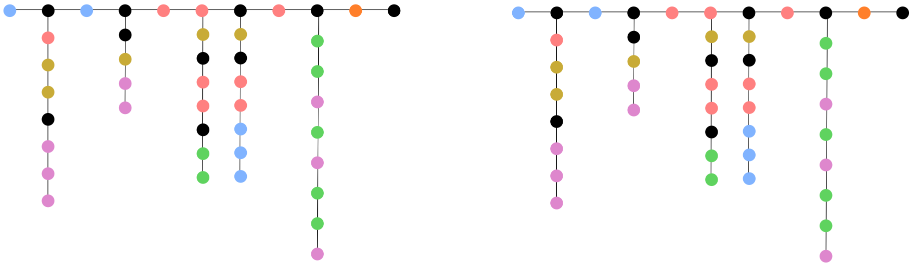


comb graphs

COLORFUL MOTIFS

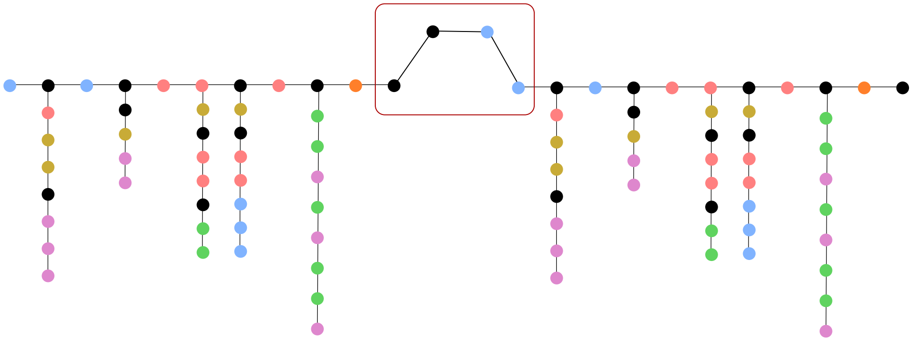
No polynomial kernels[1].

[1]Unless CoNP $\subseteq$ NP/poly



comb graphs - composition

COLORFUL MOTIFS

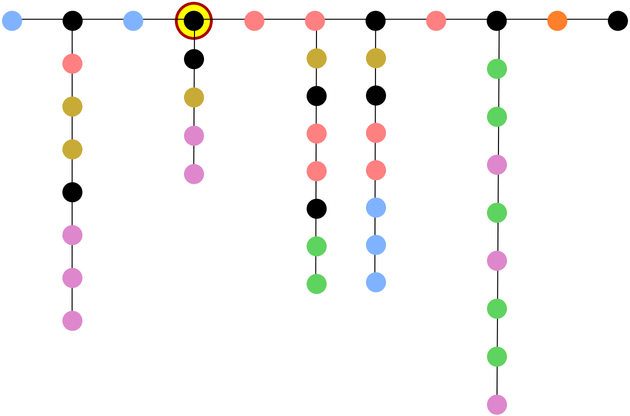


comb graphs – composition

COLORFUL MOTIFS

Many polynomial kernels.

COLORFUL MOTIFS



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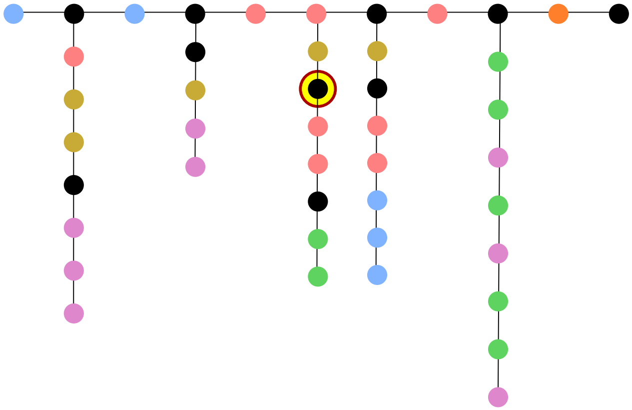

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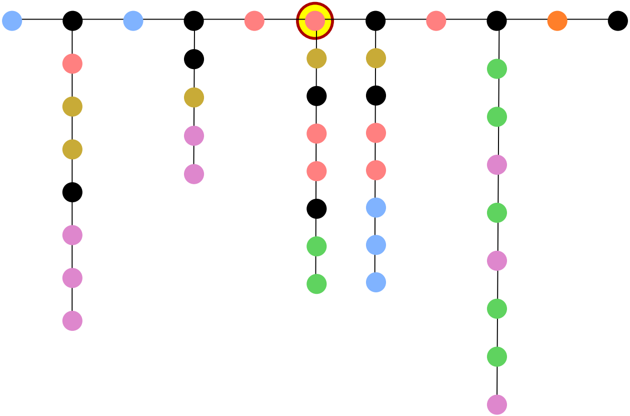

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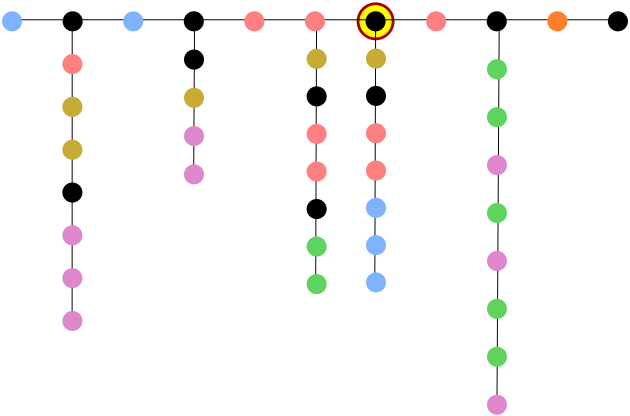

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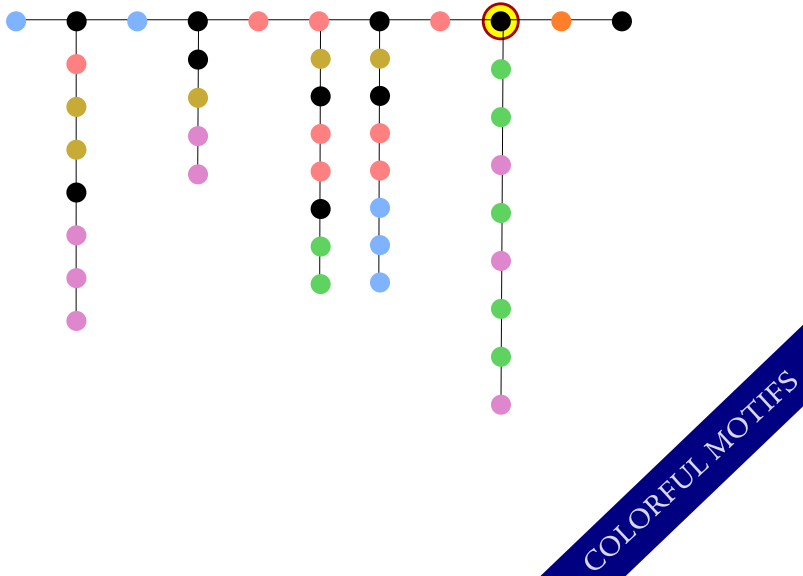
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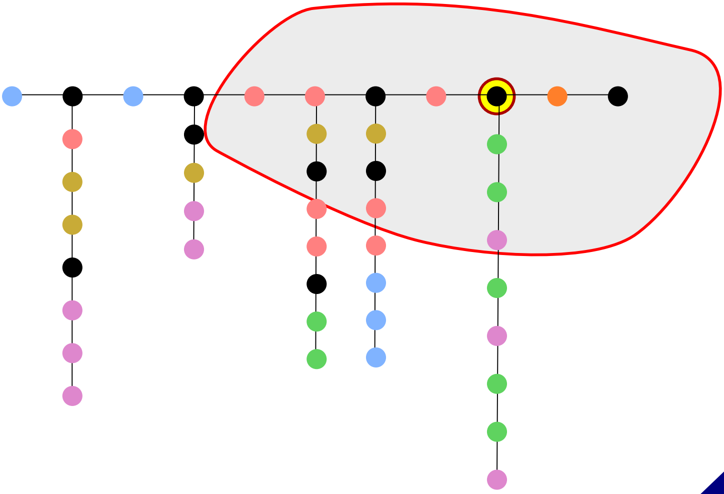

On comb graphs:

We get n kernels of size $\mathcal{O}(k^2)$ each.

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On what other classes can we obtain many polynomial kernels using this observation?

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Unfortunately, this observation does not take us very far.

Rooted Colorful Motif does not admit a polynomial kernel on trees.

“Fixing” a single vertex *does not help* the cause of kernelization.

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SUBSET COLORFUL MOTIF does not admit a polynomial kernel on trees.

"Fixing" a single vertex *does not help* the cause of kernelization.

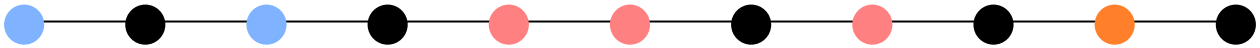
"Fixing" a constant subset of vertices does not help either.

Many polynomial kernels for trees? On more general classes of graphs?

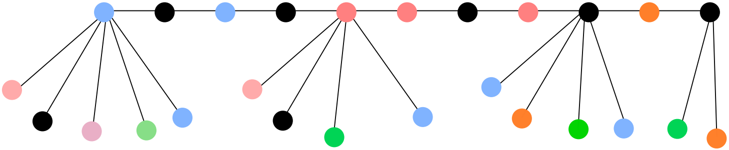
OPEN PROBLEM

A framework for ruling out the possibility of many polynomial kernels?

OPEN PROBLEM

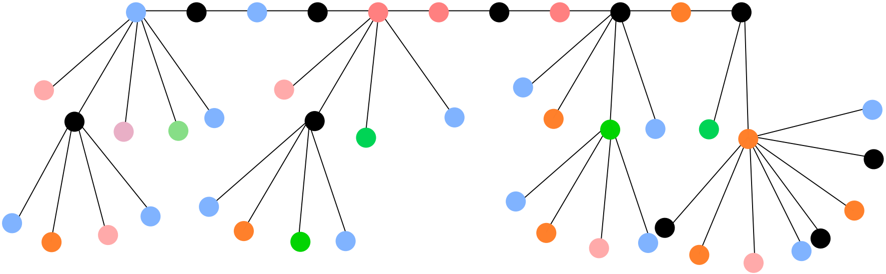


Paths: Polynomial time.



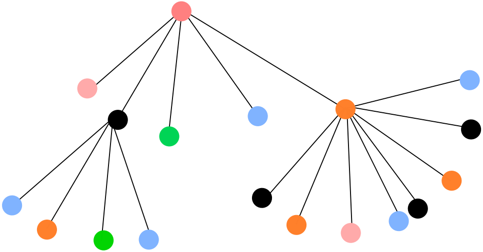
Catterpillars: Polynomial time.

COLORFUL MOTIFS



Lobsters: NP-hard even when...

COLORFUL MOTIFS



…restricted to “superstar graphs”.

COLORFUL MOTIFS

“Hardness” on superstars $\rightarrow$ “Hardness” on graphs of diameter two.

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“Hardness” on general graphs $\rightarrow$ “Hardness” on graphs of diameter three.

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NP-hardness

"Hardness" on superstars → "Hardness" on graphs of diameter two.

"Hardness" on general graphs → "Hardness" on graphs of diameter three.

NP-hardness

NP-hardness *and* infeasibility of obtaining polynomial kernels

Polynomial kernels for graphs of diameter two? For superstars?

OPEN PROBLEM

Connected Dominating Set on Graphs of Diameter One

AN APPLICATION

Connected Dominating Set on Graphs of Diameter One

Constant Time

AN APPLICATION

Connected Dominating Set on Graphs of Diameter Three

An Application

Connected Dominating Set on Graphs of Diameter Three

W[2]-hard (Split Graphs)

An Application

Connected Dominating Set on Graphs of Diameter Two

AN APPLICATION

Connected Dominating Set on Graphs of Diameter Two

???

AN APPLICATION

Connected Dominating Set on Graphs of Diameter Two

NP-complete

AN APPLICATION

Connected Dominating Set
on Graphs of Diameter Two

W[2]-hard

AN APPLICATION

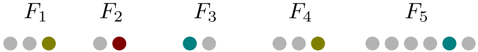
Colorful Motifs on graphs of diameter two

$\downarrow$

Connected Dominating Set on graphs of diameter two

AN APPLICATION

Reduction from Colorful Set Cover
to Colorful Motifs on Superstars



COLORFUL MOTIFS

Reduction from Colorful Set Cover
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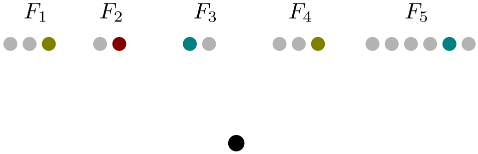

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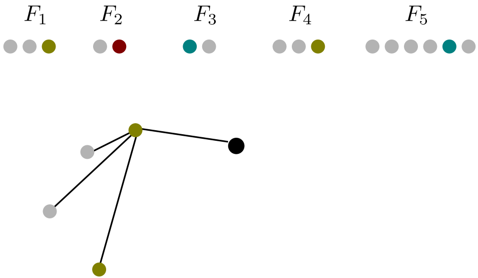

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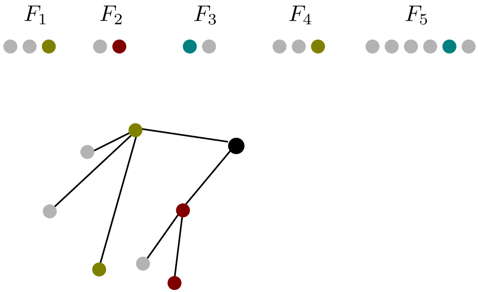

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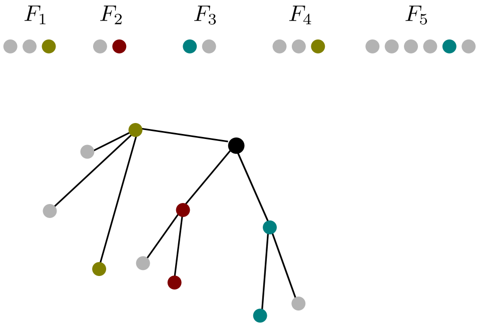

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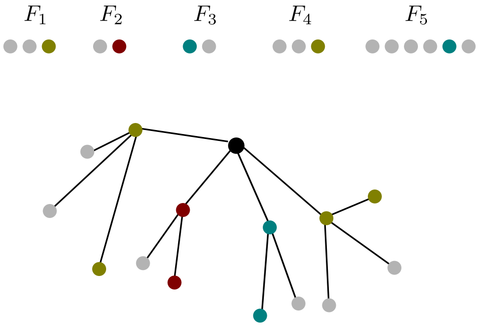

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Reduction from Colorful Set Cover
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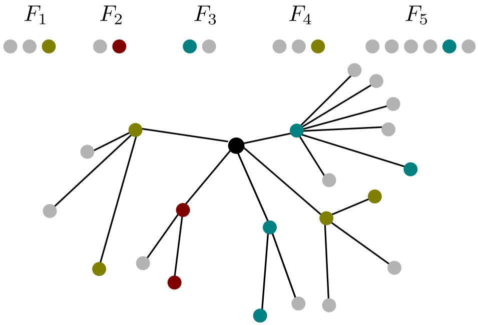

This was an advertisement for Colorful Motifs.

Concluding Remarks

This was an advertisement for Colorful Motifs.

We wish to propose that it may be a popular choice as a problem to reduce from, for showing hardness of polynomial kernelization, and even NP-hardness, for graph problems that have a connectivity requirement from the solution.

Concluding Remarks

Several open questions in the context of kernelization for the colorful motifs problem alone, and also for closely related problems.

Concluding Remarks

When we encounter hardness, we are compelled to look out for other parameters for improving the situation. There is plenty of opportunity for creativity: finding parameters that are sensible in practice *and* useful for algorithms.

Concluding Remarks

Thank you!